

## C Linear System

**Definition C.1. System:**

**Definition C.2.** A **linear system** is a system whose output is linearly related (or directly proportional) to its input<sup>28</sup>. In particular, when we say that the input and output are linearly related, we mean they need to satisfy two properties:

- (a) Homogeneous (Scaling): If the input is multiplied by a constant  $k$ , then we should observe that the output is also multiplied by  $k$ .
- (b) Additive: If the inputs are summed then the outputs are summed.

**Example C.3.** Is the function  $f(x) = x^2 + 1$  linear?

**Example C.4.** Is the function  $f(x) = 3x + 1$  linear?

**C.5.** Any **one-dimensional linear function** can be written in the form

$$y = ax$$

for some constant  $a$ .

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<sup>28</sup>The input and output are sometimes referred to as cause and effect, respectively.

- For a system, we may call it a **single-input single-output (SISO)** system.
- In radio it is the use of only one antenna both in the transmitter and receiver.

**C.6.** Any **multi-dimensional linear function** can be written in the form

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

for some matrix  $\mathbf{A}$ .

- For a system, when both  $m$  and  $n$  are greater than one, we may call it a **multiple-input multiple-output system (MIMO)** system.
- When  $m = n = 1$ , we are back to the one-dimensional case in C.5.